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The effect of lateral mass flux on the natural convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation

Mohamed E. Ali

King Saud University, Mechanical Engineering Department, PO Box 800, Riyadh 11421, Saudi Arabia

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Abstract

The effect of suction or injection on the free convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with an exponential decaying heat generation is studied. Similarity solutions are obtained for the governing steady laminar boundary layer equations using Darcy and Boussinesq approximations. The plate is assumed to have a power law temperature distribution. Three distinct cases of uniform lateral mass flux, uniform surface temperature, and uniform heat flux are studied. The effects of suction/injection parameter f_w and temperature exponent λ on the flow of heat transfer are studied. Some exact analytical results are obtained for $\lambda = 1, -1/3$. Critical values of the suction/injection parameter are obtained for adiabatic surface as a function of the temperature exponent parameter λ.

Keywords: Porous media; Similarity solution; Heat generation; Vertical plate; Natural convection

1. Introduction

Heated surfaces embedded in saturated porous medium have many geophysical and engineering applications. Such applications are flow of groundwater, geothermal energy utilization, insulation of buildings, energy storage and recovery and chemical reactor engineering.

Comprehensive reviews of the convection through porous media have reported by Nield and Bejan [1] and by Ingham and Pop [2]. Cheng and Minkowycz [3] studied the steady free convection about a vertical plate embedded in a porous media using the boundary layer assumptions and Darcy model by the similarity method. Cheng [4] extended the work of [3] by studying the effect of lateral mass flux with prescribed temperature and velocity as power law on the vertical surface. The necessary and sufficient conditions, under which similarity solutions exist for free convection boundary layers adjacent to flat plates in porous media were reported by Johnson and Cheng [5] using a power law forms. Other investigators [6–9] studied some similar porous medium cases using Darcy and Boussinesq approximations with different power law velocity and temperature variation at the boundaries. Furthermore, exact analytical solutions for free convection boundary layers on a heated vertical plate with lateral mass flux embedded in a saturated porous medium were reported by Magyari and Keller [10]. In their study exact analytical solutions are reported for some temperature exponent index $\lambda = 1, -1/3$, and $-1/2$ and they found that for $\lambda = -1/2$, solutions can only exist for suction ($f_w > 0$) and they referred to this condition as suction-born. A new class of similarity solutions has obtained for isothermal vertical plate in a semi-infinite quiescent fluid with internal heat generation decaying exponentially by Crepeau and Clarksean [11]. Postelnicu and Pop [12] have used the same source function to study the boundary layers developed by heated vertical and horizontal surfaces in porous medium with power-law wall temperature distribution. Postelnicu et al. [13] extended the work of [12] for permeable vertical surface. Free convection boundary layer developed by a vertical flat plate in porous medium, saturated with a non-Newtonian fluid with internal heat generation, was reported by Grosan and Pop [14,15]. Furthermore, similarity solution for free convection boundary layer over a non-

E-mail address: mali@ksu.edu.sa (M.E. Ali).

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Nomenclature

A	constant (>0)
C	specific heat of the fluid
f	dimensionless stream function
f_w	suction/injection parameter
g	acceleration due to gravity
k	thermal conductivity of porous medium
K	permeability of porous medium
Nu_x	local Nusselt number
q'''	internal heat generation rate per unit volume
Ra_x	modified local Rayleigh number $(gK\beta(T_w - T_\infty)x/\alpha\nu)$
T	temperature
u, v	velocity components in x and y directions
x, y	Coordinates along and normal to the plate, respectively

Greek symbols

α	the equivalent thermal diffusivity $(k/\rho C)$
β	thermal expansion coefficient
λ	temperature exponent
η	similarity variable
θ	dimensionless temperature
ρ	density
ν	kinematic viscosity
ψ	stream function

Subscripts

w	plate condition
∞	ambient condition

Superscript

$'$	differentiation with respect to η

isothermal two-dimensional or axisymmetric body embedded in a porous medium with internal heat generation is reported by Bagai [16].

The present paper studies numerically and analytically the effects of lateral mass flux on a heated vertical wall embedded in a saturated porous medium with internal heat generation for various values of temperature exponent λ.

2. Mathematical analysis

Consider the laminar steady two-dimensional motion of free convection boundary layers flow induced by a heated vertical plate embedded in a homogeneous porous medium of uniform ambient temperature T_∞ and with internal heat generation q''' as shown in Fig. 1. The equations governing this boundary layer flow using Darcy and Boussinesq approximations for incompressible viscous fluid are [1]

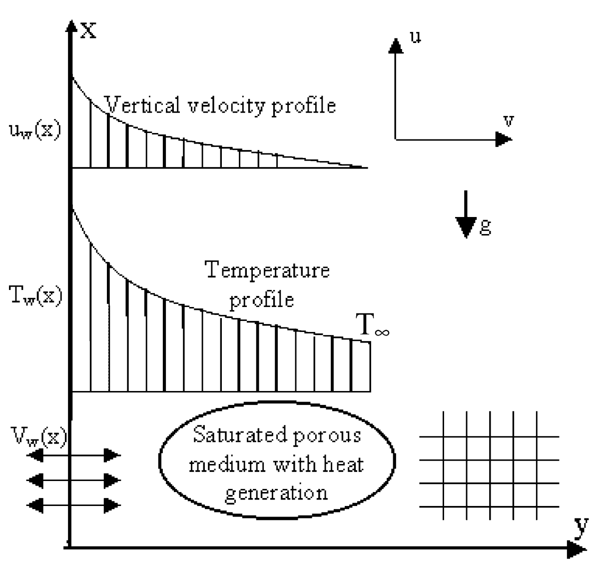

Fig. 1. Schematic of the coordinate system, boundary conditions, vertical velocity and temperature profiles adjacent to a permeable heated vertical surface embedded in a saturated porous medium with internal heat generation.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} = \frac{gK\beta}{\nu}\frac{\partial T}{\partial y} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{q'''}{\rho C} \tag{3}$$

Symbols and their definitions are given in the nomenclature and ρC is the heat capacity per unit volume of the fluid. It is also assumed that the temperature distribution of the plate is governed by the power law $T_w(x) = T_\infty + Ax^\lambda$, where T_∞ is the temperature at infinity and A is a constant > 0 for heated plate.

Eqs. (1)–(3) are subject to the following boundary conditions:

$$T(x,0) = T_w(x) \quad \text{and} \quad v(x,0) = v_w(x)$$
$$T(x,\infty) = T_\infty \quad \text{and} \quad u(x,\infty) = 0 \tag{4}$$

where the Cartesian coordinates x and y are measured along the plate and normal to it respectively (see Fig. 1). Following Postelnicu and Pop [12], if the heat generation rate is of the form

$$q''' = \frac{k(T_w - T_\infty)}{x^2} Ra_x e^{-\eta} \tag{5}$$

then Eqs. (1)–(4) admit the following similarity solution

$$\psi = \alpha Ra_x^{1/2} f(\eta), \qquad \eta = Ra_x^{1/2}\left(\frac{y}{x}\right),$$
$$u = \left(\frac{\alpha}{x}\right) Ra_x f'(\eta), \qquad T = T_\infty + Ax^\lambda \theta(\eta),$$
$$\theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$v = -\left(\frac{\alpha}{2x}\right) Ra_x^{1/2}\left[(\lambda+1)f + (\lambda-1)\eta f'\right] \tag{6}$$

where f, f', and θ are the dimensionless stream function, vertical velocity, and temperature field respectively and $Ra_x = gK\beta(T_w - T_\infty)x/(\alpha\nu)$ is the modified local Rayleigh number. Substitution in the governing equations (1)–(3) gives rise to the following system of ordinary differential equations

$$f'' = \theta' \tag{7}$$

$$\theta'' + \frac{\lambda+1}{2} f\theta' - \lambda f'\theta + e^{-\eta} = 0 \tag{8}$$

and are subject to the following boundary conditions:

$$\theta(0) = 1, \qquad \theta(\infty) = 0 \tag{9}$$

$$f(0) = f_w, \qquad f'(\infty) = 0 \tag{10}$$

The suction or injection speed v_w at the wall is

$$v_w = -\left(\frac{\alpha}{2x}\right) Ra_x^{1/2}(\lambda+1) f(0) \tag{11}$$

The quantity $f(0) = f_w$ is referred to as the dimensionless suction/injection parameter. Therefore, $f_w =$ zero corresponding to an impermeable surface where the equations reduce to those of Postelnicu and Pop [12] and of course their solutions are recoverable. On the other hand, if the internal heat generation is off, the equations reduce to those of Cheng and Minkowycz [3], and Magyari and Keller [10]. Furthermore, the plate is permeable with suction or injection according to $f_w > 0$ or $f_w < 0$ respectively.

Eq. (7) and the boundary conditions (9)–(10) yield that, $f'(\eta) = \theta(\eta)$ which shows [3,4,10,12] that the dimensionless vertical velocity and temperature profiles are identical. Therefore, according to Eqs. (7)–(11), the present problem reduces to the solution of the following nonlinear ordinary differential equation

$$f''' + \frac{\lambda+1}{2} ff'' - \lambda f'^2 + e^{-\eta} = 0 \tag{12}$$

subject to the boundary conditions

$$f'(0) = 1, \qquad f(0) = f_w, \qquad f'(\infty) = 0 \tag{13}$$

Eqs. (12)–(13) are compatible with those given by [13]. The local surface heat flux is then, given by:

$$q_w(x) = -kA\left(\frac{g\beta KA}{\alpha\nu}\right)^{1/2} x^{\frac{3\lambda-1}{2}}\theta'(0) \tag{14}$$

and it can be expressed as a function of the local Rayleigh and Nusselt numbers as

$$Nu_x Ra_x^{-1/2} = -\theta'(0) \tag{15}$$

and the entrainment velocity of the fluid is given by

$$v(x,\infty) = -\left(\frac{\alpha}{2x}\right) Ra_x^{1/2}(\lambda+1) f(\infty) \tag{16}$$

Following Magyari and Keller [10] integrating Eq. (12) across the boundary layer from zero to ∞ using Eq. (7) and the boundary conditions (13) leads to for the dimensionless surface heat flow

$$\frac{Nu_x}{Ra_x^{1/2}} = \left(\frac{\lambda+1}{2}\right) f_w + \left(\frac{3\lambda+1}{2}\right)\int_0^\infty f'^2(\eta)\,d\eta - 1 = -\theta'(0) \tag{17}$$

2.1. Exact analytical solution for $\lambda = 1$ and $f_w = 1$

The problem (7)–(10) admits the exact analytical solution

$$f(\eta) = 2 - e^{-\eta}, \qquad \theta(\eta) = e^{-\eta} \tag{18}$$

such that

$$f''(0) = \theta'(0) = -1 \quad \text{and} \tag{19}$$

$$f(\infty) = 2 \tag{20}$$

This case is confirmed computationally where $f''(0) = \theta'(0) = -0.99996$ and $f(\infty) = 1.9997$ when $\eta_{max} = 15$ with a step of 0.0001.

2.2. Consequences of the integral relationship (17)

The following remarks can be observed:

First: for $\lambda = -1/3$ and $f_w = 0$

Eq. (17) admits an exact analytic result as:

$$f''(0) = \theta'(0) = -Nu_x Ra_x^{-1/2} = 1 \quad (\lambda = -1/3,\ f_w = 0) \tag{21}$$

This analytical result agrees very good with the computational result obtained for this case as seen in Table 1.

Second: for $\lambda = -1/3$ one obtains for $\theta'(0)$ and f_w the explicit relationship

$$\theta'(0) = 1 - \frac{1}{3} f_w \quad (\lambda = -1/3) \tag{22}$$

Third: Since the integral in (17) is always positive, one can conclude the following inequality for $\lambda \neq -1/3$

$$\frac{1 - \frac{\lambda+1}{2} f_w - \theta'(0)}{3\lambda+1} > 0 \quad (\lambda \neq -1/3) \tag{23}$$

Therefore, from Eq. (23) one can obtains that

$$\theta'(0) < 1 - \frac{\lambda+1}{2} f_w \quad \text{for } \lambda > -1/3 \tag{24}$$

and

$$\theta'(0) > 1 - \frac{\lambda+1}{2} f_w \quad \text{for } \lambda < -1/3 \tag{25}$$

respectively.

Fourth: The adiabatic surface ($\theta'(0) = 0$) condition

Table 1
Comparison with the previously published results for impermeable heated vertical plate embedded in porous medium with internal heat generation q'''

λ	$f''(0) = \theta'(0) = -Nu_x Ra_x^{-1/2}$		
	Postelnicu and Pop [12]	Bagai [16]	Present results
−1/3	0.99961	–	1.000501
−1/4	0.67917	–	0.67985
0	0.21524	0.21524	0.21566
1/3	−0.11415	−0.11415	−0.114075
1	−0.52409	−0.52409	−0.523878

Table 2
Critical values of the suction/injection parameter $(f_w)_c$ where the surface is adiabatic for various values of λ showing the temperature gradients at the surface $\theta'(0)$ that are almost zero

λ	$(f_w)_c$	$\theta'(0) = -Nu_x Ra_x^{-1/2}$
−0.5	4.95214	0.0000039
−1/3	3.0000	0.0000954
−0.25	2.32415	0.0000077
0	0.86793	0.0000069
0.15	0.20464	0.0000072
1/3	−0.5441	0.0000042
0.5	−1.2945	0.0000088

The adiabatic surface case can be realized as follows:

- only for $f_w = 3$ when $\lambda = -1/3$ as a result of Eq. (22) (26)
- only for a certain $f_w < 2/(\lambda + 1)$ when $\lambda > -1/3$ (from Eq. (24)) (27)
- only for a certain $f_w > 2/(\lambda + 1)$ when $-1 < \lambda < -1/3$ (from Eq. (25)) (28)

The adiabatic surface conditions are calculated computationally and checked with the above three conditions as seen in Table 2.

3. Numerical solution procedure

The nonlinear equation (12) subject to the boundary conditions (13) is solved numerically by using the fourth order Runge–Kutta method. Solutions are obtained for different values of f_w for constant temperature exponent λ. Computations were started from a known solution for f_w = zero with internal heat generation following Postelnicu and Pop [12] or for zero heat generation following Cheng [4] and Magyari and Keller [10]. For a given value of f_w, at constant λ, the value of $f''(0)$ is estimated and the differential equation (12) is integrated until the boundary condition at infinity $f'(\infty)$ is satisfied by decaying exponentially to zero. If the boundary condition at infinity is not satisfied then the numerical routine uses a half interval method to calculate corrections to the estimated value of $f''(0)$. The value of η_{max} was chosen as large as possible depending on the dimensionless suction/injection parameter f_w without causing numerical oscillations in the values of f'. Comparisons were made with Postelnicu and Pop [12] and Bagai [16] for impermeable plate with heat generation in Table 1 and with Cheng [4] and Magyari and Keller [10] for permeable plate with no heat generation in Table 3 for $\lambda = 1$, which show good agreements. Good comparisons were also obtained with Postelnicu et al. [13] for permeable surface with heat generation as seen in Table 4.

Table 3
Comparison with the previously published results for heated vertical plate embedded in porous medium for $\lambda = 1$ with lateral mass flux and no heat generation

f_w	$f''(0) = \theta'(0) = -Nu_x Ra_x^{-1/2}$		
	Magyari and Keller [10]	Cheng [4]	Present results
−1.0	−0.6180	−0.6180	−0.61803
−0.8	−0.6770	−0.6770	−0.67703
−0.4	−0.8198	−0.8198	−0.81980
0.0	−1.0000	−1.0000	−1.00000
1.0	−1.6180	−1.6180	−1.61803

4. Results and discussion

The governing equation (12) subject to the boundary conditions (13) is integrated as described in Section 3. Solutions are obtained for three distinct values of the temperature exponent parameter $\lambda = 0, 1$, and $1/3$ which corresponding to isothermal plate, uniform lateral mass flux at the plate, and uniform heat flux independent of x but surface temperature gradient dependent only respectively. However, other values of λ are also considered such as $\lambda = -0.5, -0.25, 1/2$, and $-1/3$ to complete the picture of the problem.

Fig. 2 shows the velocity $f'(\eta)$ or temperature $\theta(\eta)$ profiles across the boundary layers for $\lambda = 0$ and different values of the suction/injection parameter f_w. As mentioned earlier suction corresponding to $f_w > 0$, injection to $f_w < 0$, and $f_w = 0$ to impermeable plate, therefore it is clear that suction reduces the boundary layer thickness sharply as seen for $f_w = 50$ while injection increases it as for $f_w = -2$. It should be mentioned that the critical value where the surface is almost adiabatic is occurred at $(f_w)_c = 0.86793$ as given in Table 2. Furthermore, for any $f_w > (f_w)_c$ (suction) the surface heat flow is always positive and it is directed from the plate to the porous medium. On the other hand, the opposite is true for $f_w < (f_w)_c$ where heat is transferred from the porous medium to the plate.

The uniform lateral mass flux case ($\lambda = 1$) presented by Eq. (11) using the definition of Rayleigh number where v_w is uniform along the plate (not function of x but f_w dependent only) is shown in Fig. 3. In this figure, the surface heat flow is always positive regardless of the sign of f_w where the

Table 4
Comparison with the previously published results for permeable heated vertical plate embedded in porous medium with internal heat generation q'''

f_w	$f''(0) = \theta'(0) = -Nu_x Ra_x^{-1/2}$					
	$\lambda = 0$		$\lambda = 1/3$		$\lambda = 1.0$	
	Present results	Postelnicu et al. [13]	Present results	Postelnicu et al. [13]	Present results	Postelnicu et al. [13]
−1.0	0.3658	0.3654	0.06636	0.0662	−0.2550	−0.2550
−0.6	0.3184	0.3182	0.0095	0.0094	−0.3407	−0.3407
0.6	0.0744	0.0742	−0.2869	−0.2869	−0.7837	−0.7837
1.0	−0.0387	−0.0391	−0.4288	−0.4289	−0.9999	1.0000

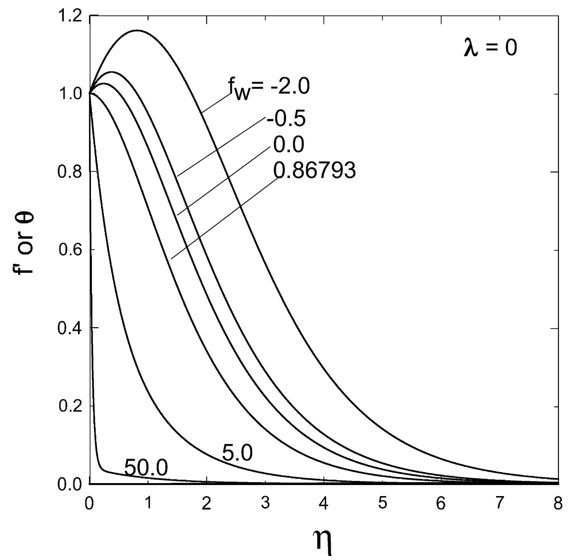

Fig. 2. Vertical velocity or temperature profiles for uniform wall temperature ($\lambda = 0$) for different values of suction/injection f_w.

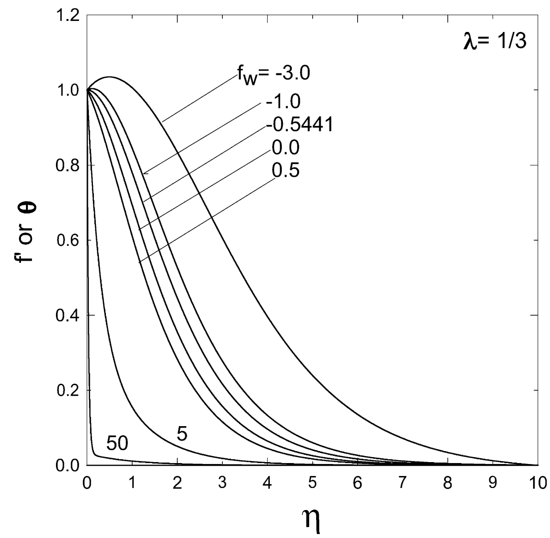

Fig. 4. Vertical velocity or temperature profiles for uniform heat flux ($\lambda = 1/3$) for different values of suction/injection f_w.

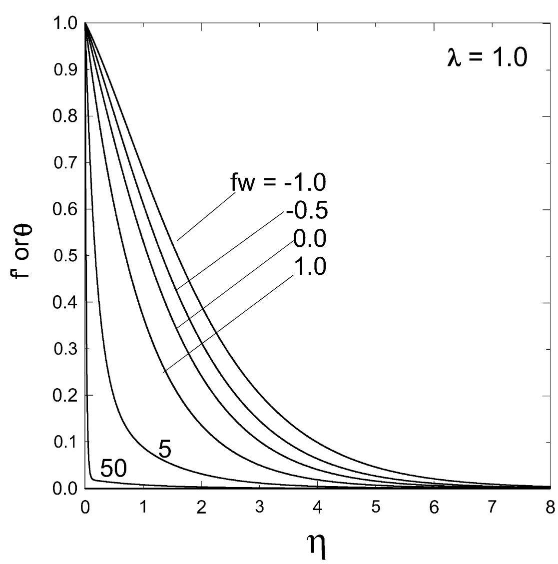

Fig. 3. Vertical velocity or temperature profiles for uniform lateral mass flux ($\lambda = 1$) for different values of suction/injection f_w.

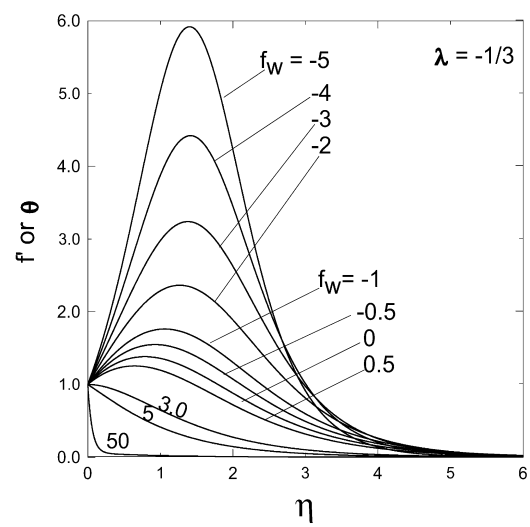

Fig. 5. Vertical velocity or temperature profiles for $\lambda = -1/3$ for different values of suction/injection f_w. $f_w = 3.0$ corresponds to adiabatic and frictionless ($f''(0) = \theta'(0) = 0$) slipping of the fluid along the wall.

heat is directed from the plate to the porous medium. Consequently, the boundary layer thickness decreases as f_w increases. It should be mentioned that the case of $f_w = 1$ which is obtained analytically by Eqs. (18)–(20) is confirmed here as mentioned earlier. On the other hand, the uniform heat flux case obtained by Eq. (14) for $\lambda = 1/3$ where q_w is not function of x is demonstrated in Fig. 4 in terms of the temperature profile for various f_w. As in the previous figures the boundary layer thickness decreases as f_w increases and the surface heat transfer is directed from the surface to the convecting fluid for $f_w > (f_w)_c = -0.5441$, where at this critical value the surface is adiabatic. However, for $f_w < (f_w)_c$ heat is directed towards the surface from the porous medium. Furthermore, the inequality (24) is satisfied as well as in the previous Figs. 2 and 3 since $\lambda > -1/3$. It should be noted that, other critical values corresponding to different values of λ are reported in Table 2 and plotted in Fig. 7.

Other interesting values to show the effect of suction or injection on the temperature profiles across the boundary layers for $\lambda = -1/3$ are reported in Fig. 5. In this figure for $f_w > (f_w)_c = 3.0$ heat is transferred from the wall to the porous medium and for $f_w < (f_w)_c$, heat is reversed and transferred to the wall from the porous medium. It should be noted that the adiabatic condition in this case no longer occurs at $f_w = 0$ due to the internal heat generation included in the present analysis. Furthermore, The changes in the temperature profiles are marked by the occurrence of a hill for $f_w < (f_w)_c$. As f_w decreases from $(f_w)_c$ to $-\infty$, the height of the hill (velocity overshoot) approaches infinity similar to the case of no heat generation (see Magyari and Keller [10]). Also here the analyt-

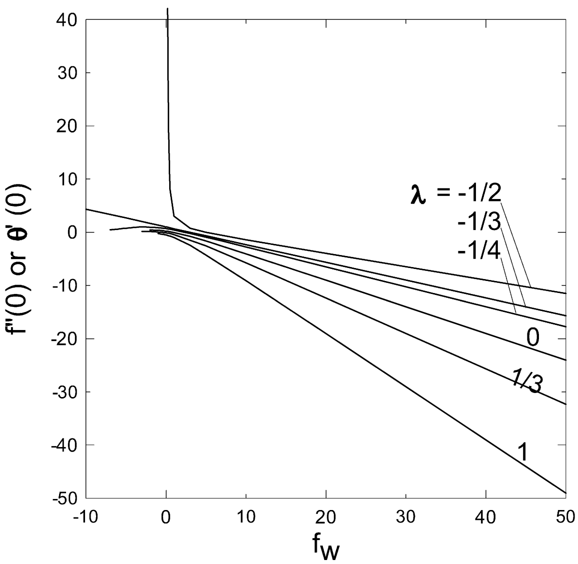

Fig. 6. Dimensionless shear stress or temperature gradients at the wall for different λ showing the case of $\lambda = -\frac{1}{2}$ where the solution does not exist as f_w approaches zero.

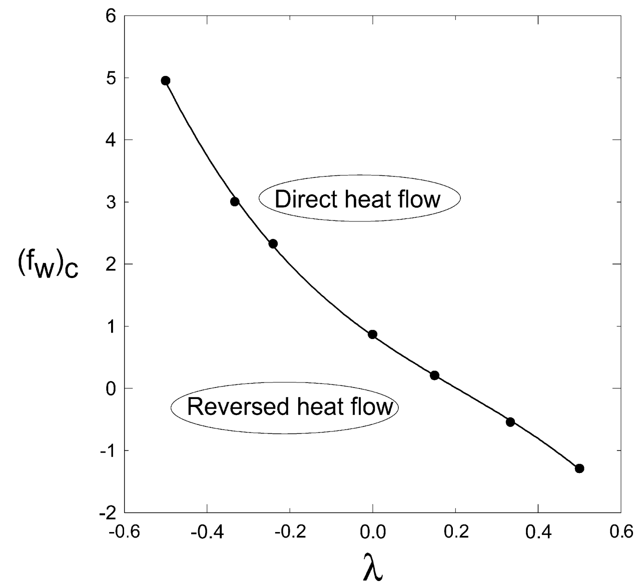

Fig. 7. Critical values of the suction/injection parameter $(f_w)_c$ where the surface is adiabatic. Table 3 gives the point coordinates, while Eq. (29) gives the fitting curve.

ical conditions (21) and (22) are confirmed as seen in Table 1 and 2 for $f_w = 0$ and 3, respectively.

The temperature gradients at the surface $\theta'(0)$ are plotted against f_w in Fig. 6 for various values of λ. It is clear that, for $f_w > 0$ (suction) the temperature gradient at the surface increases as λ decreases and from Eq. (15) one can see that $Nu_x Ra_x^{-1/2}$ has an opposite sense in other words, it increases with increasing λ. It is interesting here to observe that for $\lambda = -1/2$ as f_w approaches zero there is no solution can be obtained and this observation agrees with that of impermeable wall by Ingham and Brown [9] and by Magyari and Keller [10] for no heat generation. However, as f_w increases (suction) the solution can be obtained and it was called "suction-born" flow [10]. Furthermore, for this kind of boundary layer flow the adiabatic wall condition occurs at $f_w = (f_w)_c = 4.95214$ and for $f_w > (f_w)_c$ heat is transferred from the surface to the porous medium whereas for $(f_w)_c > f_w \geqslant 0.2$ the surface heat flow is reversed. It should be noted that, for $\lambda = -1/2$ the analytical inequality (25) is satisfied and confirmed computationally.

Fig. 7 summarizes the heat transfer flow where $(f_w)_c$ is plotted verses λ. In this figure, above the solid curve $(f_w > (f_w)_c)$ heat is transferred from the plate to the porous medium where $\theta'(0)$ is negative and $Nu_x Ra_x^{-1/2}$ is positive (direct heat flow). However, below the curve $(f_w < (f_w)_c)$ heat is transferred from the porous medium to the plate where $\theta'(0)$ is positive and $Nu_x Ra_x^{-1/2}$ is negative (reversed heat flow). Furthermore, on the solid line $(f_w = (f_w)_c)$ the temperature gradient at the surface $\theta'(0) = 0$ where the surface is adiabatic according to Table 2. According to Eq. (14) the heat flux is vanishing for $\theta'(0) = 0$ in every point of the surface except for the leading edge singularity at $x = 0$ where the heat responsible for the temperature field is released by this singularity. It should be noted, that the coordinates of the points in Fig. 7 are given in Table 2 and the polynomial fitting curve through the points is given by (using Golden Software Grapher)

$$(f_w)_c = 0.846 - 4.705\lambda + 3.867\lambda^2 - 6.134\lambda^3$$
$$0.5 \geqslant \lambda \geqslant -0.5 \tag{29}$$

where the polynomial correlation coefficient is 99.97%. Finally, it should be observed from Fig. 7 and Table 2 that for $\lambda = -1/3$, $> -1/3$ and $< -1/3$ the analytical conditions obtained by (26), (27), and (28) are satisfied respectively.

5. Conclusions

Similarity solutions are obtained for the governing equations (12)–(13). Special cases are considered for the plate temperature exponent λ with lateral mass flux controlled by the suction/injection parameter f_w. The case $\lambda = 0$ corresponds to a uniform surface temperature where heat is transferred from the plate to the convecting fluid for $f_w < 0.86793$ while for $f_w > 0.86793$ heat is reversed and transferred from the porous medium to the plate. The uniform lateral mass flux is presented by $\lambda = 1$ where for all $50.0 \geqslant f_w \geqslant -1.0$ heat is transferred in a direct way from the plate to the medium with positive $Nu_x Ra_x^{-1/2}$ or negative temperature gradient at the wall. Furthermore, $\lambda = 1/3$ corresponding to uniform heat flux at the plate independent of x but temperature gradient at the wall dependant only, where the adiabatic surface condition occurs at $(f_w)_c = -0.5441$. Moreover, solutions for $\lambda = -1/3$ show that as the injection strength increases the temperature profile has a hill (velocity overshoot) where the heat flow is directed in a reverse way. On the other hand, $\lambda = -\frac{1}{2}$ corresponds to similar boundary flow, which only exist for suction case $(f_w > 0.2)$ and as f_w approaches zero no solutions are obtained similar to the case with no heat generation reported by [10]. Analytical solutions and conditions obtained in Sections 2.1 and 2.2 for different values of λ are confirmed computationally. Finally, in all cases, boundary layer thickness reduces by suction and increases by injection with critical plate adiabatic conditions given in Table 2.

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